Then, the intensity is

$$I = I_0 \left(\frac{\sin \beta}{\beta}\right)^2 = I_0 \left(\frac{\sin(\pi/3)}{\pi/3}\right)^2 = 0.684 I_0.$$

Significance

Note that this approach is relatively straightforward and gives a result that is almost exactly the same as the more complicated analysis using phasors to work out the intensity values of the double-slit interference (thin line in **Figure 4.11**). The phasor approach accounts for the downward slope in the diffraction intensity (blue line) so that the peak *near* $m = 1$ occurs at a value of θ ever so slightly smaller than we have shown here.

Example 4.4

Two-Slit Diffraction

Suppose that in Young's experiment, slits of width 0.020 mm are separated by 0.20 mm. If the slits are illuminated by monochromatic light of wavelength 500 nm, how many bright fringes are observed in the central peak of the diffraction pattern?

Solution

From **Equation 4.1**, the angular position of the first diffraction minimum is $\theta \approx \sin\theta = \frac{\lambda}{D} = \frac{5.0 \times 10^{-7}\ \text{m}}{2.0 \times 10^{-5}\ \text{m}} = 2.5 \times 10^{-2}\ \text{rad}.$

Using $\sin\theta = m\lambda$ for $\theta = 2.5 \times 10^{-2}\ \text{rad}$, we find

$$m = \frac{d \sin\theta}{\lambda} = \frac{(0.20\ \text{mm})\left(2.5 \times 10^{-2}\ \text{rad}\right)}{\left(5.0 \times 10^{-7}\ \text{m}\right)} = 10,$$

which is the maximum interference order that fits inside the central peak. We note that $m = \pm 10$ are missing orders as θ matches exactly. Accordingly, we observe bright fringes for

$$m = -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, \text{ and } +9$$

for a total of 19 bright fringes.

4.3 Check Your Understanding For the experiment in **Example 4.4**, show that $m = 20$ is also a missing order.

Explore the effects of double-slit diffraction. In **this simulation (https://openstaxcollege.org/l/21doubslitdiff)** written by Fu-Kwun Hwang, select $N = 2$ using the slider and see what happens when you control the slit width, slit separation and the wavelength. Can you make an order go "missing?"

4.4 | Diffraction Gratings

Learning Objectives

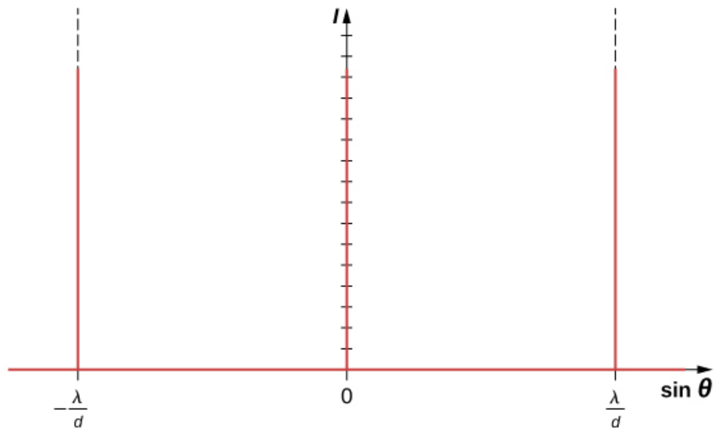
By the end of this section, you will be able to:

- Discuss the pattern obtained from diffraction gratings
- Explain diffraction grating effects

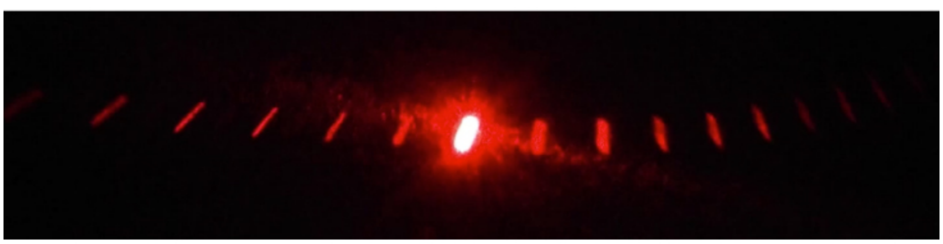
Analyzing the interference of light passing through two slits lays out the theoretical framework of interference and gives us a historical insight into Thomas Young's experiments. However, most modern-day applications of slit interference use not just two slits but many, approaching infinity for practical purposes. The key optical element is called a diffraction grating, an important tool in optical analysis.

Diffraction Gratings: An Infinite Number of Slits

The analysis of multi-slit interference in **Interference** allows us to consider what happens when the number of slits N approaches infinity. Recall that $N - 2$ secondary maxima appear between the principal maxima. We can see there will be an infinite number of secondary maxima that appear, and an infinite number of dark fringes between them. This makes the spacing between the fringes, and therefore the width of the maxima, infinitesimally small. Furthermore, because the intensity of the secondary maxima is proportional to $1/N^2$, it approaches zero so that the secondary maxima are no longer seen. What remains are only the principal maxima, now very bright and very narrow (**Figure 4.12**).



(a)



(b)

Figure 4.12 (a) Intensity of light transmitted through a large number of slits. When N approaches infinity, only the principal maxima remain as very bright and very narrow lines. (b) A laser beam passed through a diffraction grating. (credit b: modification of work by Sebastian Stapelberg)

In reality, the number of slits is not infinite, but it can be very large—large enough to produce the equivalent effect. A prime example is an optical element called a **diffraction grating**. A diffraction grating can be manufactured by carving glass with a sharp tool in a large number of precisely positioned parallel lines, with untouched regions acting like slits (**Figure 4.13**). This type of grating can be photographically mass produced rather cheaply. Because there can be over 1000 lines per millimeter across the grating, when a section as small as a few millimeters is illuminated by an incoming ray, the number of illuminated slits is effectively infinite, providing for very sharp principal maxima.

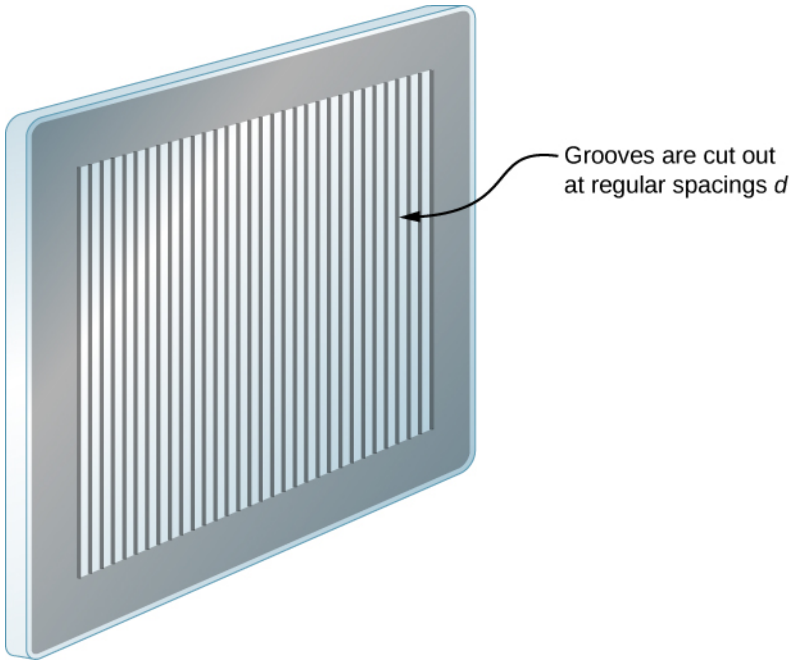


Figure 4.13 A diffraction grating can be manufactured by carving glass with a sharp tool in a large number of precisely positioned parallel lines.

Diffraction gratings work both for transmission of light, as in **Figure 4.14**, and for reflection of light, as on butterfly wings and the Australian opal in **Figure 4.15**. Natural diffraction gratings also occur in the feathers of certain birds such as the hummingbird. Tiny, finger-like structures in regular patterns act as reflection gratings, producing constructive interference that gives the feathers colors not solely due to their pigmentation. This is called iridescence.

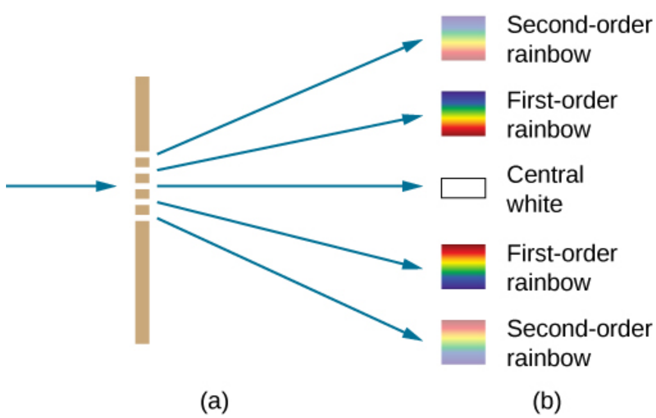


Figure 4.14 (a) Light passing through a diffraction grating is diffracted in a pattern similar to a double slit, with bright regions at various angles. (b) The pattern obtained for white light incident on a grating. The central maximum is white, and the higher-order maxima disperse white light into a rainbow of colors.

(a) (b)

Figure 4.15 (a) This Australian opal and (b) butterfly wings have rows of reflectors that act like reflection gratings, reflecting different colors at different angles. (credit a: modification of work by "Opals-On-Black"/Flickr; credit b: modification of work by “whologwhy”/Flickr)

Applications of Diffraction Gratings

Where are diffraction gratings used in applications? Diffraction gratings are commonly used for spectroscopic dispersion and analysis of light. What makes them particularly useful is the fact that they form a sharper pattern than double slits do. That is, their bright fringes are narrower and brighter while their dark regions are darker. Diffraction gratings are key components of monochromators used, for example, in optical imaging of particular wavelengths from biological or medical samples. A diffraction grating can be chosen to specifically analyze a wavelength emitted by molecules in diseased cells in a biopsy sample or to help excite strategic molecules in the sample with a selected wavelength of light. Another vital use is in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths. A range of diffraction gratings are available for selecting wavelengths for such use.

Example 4.5

Calculating Typical Diffraction Grating Effects

Diffraction gratings with 10,000 lines per centimeter are readily available. Suppose you have one, and you send a beam of white light through it to a screen 2.00 m away. (a) Find the angles for the first-order diffraction of the shortest and longest wavelengths of visible light (380 and 760 nm, respectively). (b) What is the distance between the ends of the rainbow of visible light produced on the screen for first-order interference? (See **Figure 4.16**.)

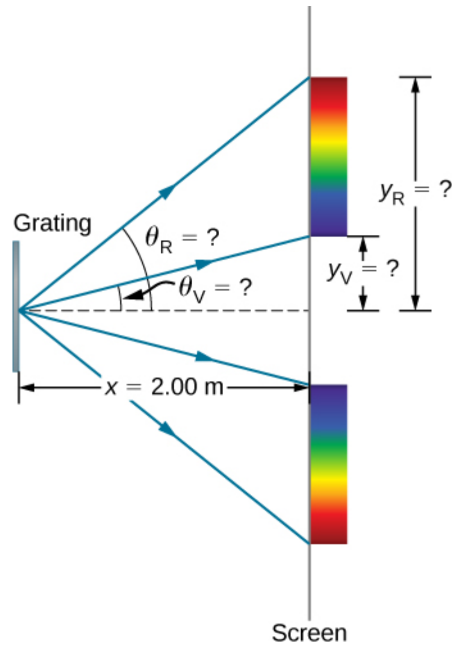


Figure 4.16 (a) The diffraction grating considered in this example produces a rainbow of colors on a screen a distance $x = 2.00\ \text{m}$ from the grating. The distances along the screen are measured perpendicular to the *x*-direction. In other words, the rainbow pattern extends out of the page.
(b) In a bird's-eye view, the rainbow pattern can be seen on a table where the equipment is placed.

Strategy

Once a value for the diffraction grating's slit spacing *d* has been determined, the angles for the sharp lines can be found using the equation

$$d \sin \theta = m\lambda \text{ for } m = 0,\ \pm 1,\ \pm 2,\ \ldots .$$

Since there are 10,000 lines per centimeter, each line is separated by 1/10,000 of a centimeter. Once we know the angles, we an find the distances along the screen by using simple trigonometry.

Solution

a. The distance between slits is $d = (1\ \text{cm})/10,000 = 1.00 \times 10^{-4}$ cm or 1.00×10^{-6} m. Let us call the two angles θ_V for violet (380 nm) and θ_R for red (760 nm). Solving the equation $d \sin \theta_V = m\lambda$ for $\sin \theta_V$,

$$\sin \theta_V = \frac{m\lambda_V}{d},$$

where $m = 1$ for the first-order and $\lambda_V = 380\ \text{nm} = 3.80 \times 10^{-7}$ m. Substituting these values gives

$$\sin \theta_V = \frac{3.80 \times 10^{-7}\ \text{m}}{1.00 \times 10^{-6}\ \text{m}} = 0.380.$$

Thus the angle θ_V is

$$\theta_V = \sin^{-1} 0.380 = 22.33°.$$

Similarly,

$$\sin\theta_{\text{R}} = \frac{7.60 \times 10^{-7}\text{ m}}{1.00 \times 10^{-6}\text{ m}} = 0.760.$$

Thus the angle θ_{R} is

$$\theta_{\text{R}} = \sin^{-1} 0.760 = 49.46°.$$

Notice that in both equations, we reported the results of these intermediate calculations to four significant figures to use with the calculation in part (b).

b. The distances on the secreen are labeled y_{V} and y_{R} in **Figure 4.16**. Notice that $\tan\theta = y/x$. We can solve for y_{V} and y_{R}. That is,

$$y_{\text{V}} = x\tan\theta_V = (2.00\text{ m})(\tan 22.33°) = 0.815\text{ m}$$

and

$$y_{\text{R}} = x\tan\theta_R = (2.00\text{ m})(\tan 49.46°) = 2.338\text{ m}.$$

The distance between them is therefore

$$y_{\text{R}} - y_{\text{V}} = 1.523\text{ m}.$$

Significance

The large distance between the red and violet ends of the rainbow produced from the white light indicates the potential this diffraction grating has as a spectroscopic tool. The more it can spread out the wavelengths (greater dispersion), the more detail can be seen in a spectrum. This depends on the quality of the diffraction grating—it must be very precisely made in addition to having closely spaced lines.

4.4 Check Your Understanding If the line spacing of a diffraction grating d is not precisely known, we can use a light source with a well-determined wavelength to measure it. Suppose the first-order constructive fringe of the H_β emission line of hydrogen $(\lambda = 656.3\text{ nm})$ is measured at $11.36°$ using a spectrometer with a diffraction grating. What is the line spacing of this grating?

Take **the same simulation (https://openstaxcollege.org/l/21doubslitdiff)** we used for double-slit diffraction and try increasing the number of slits from $N = 2$ to $N = 3, 4, 5...$. The primary peaks become sharper, and the secondary peaks become less and less pronounced. By the time you reach the maximum number of $N = 20$, the system is behaving much like a diffraction grating.

4.5 | Circular Apertures and Resolution

Learning Objectives

By the end of this section, you will be able to:

- Describe the diffraction limit on resolution
- Describe the diffraction limit on beam propagation

Light diffracts as it moves through space, bending around obstacles, interfering constructively and destructively. This can be used as a spectroscopic tool—a diffraction grating disperses light according to wavelength, for example, and is used to produce spectra—but diffraction also limits the detail we can obtain in images.